for slots which are small compared to the free-space wavelength. The reaction method was originally propounded by Das and Sanyal [11] as a means of analyzing long slots ($l>\lambda/2$). The suggested trial function for the E-field in the slot, while obviously appropriate for slots in this category, appears to be seriously in error for short slots. This is supported by Vu Khac's computations, which show [7] that for short nonresonant slots the aperture field is almost perfectly cosinusoidal. In addition, the effect of the side wall on the energy stored inside the waveguide is ignored in this "reaction" method, and it is suggested that for small slots which are almost purely reactive this could be a significant omission.

It is perhaps pertinent to point out here that the specific suggestion, made in [10], that the variational method of [5] is significantly in error near resonance, is patently incorrect. The apparent discrepancy detected by Pandharipande can be traced to an unfortunate printing error in the published paper. This error can be isolated without difficulty by performing a dimensional check on eq. (34), or by consulting fig. 4 of [5] which was extracted directly from the original thesis describing the variational method [12]. Equation (34) of [5] should read

$$c_{10}=-d_{10}=\frac{AP(I_1+h_1J_1)^2}{(X+jY)(I_1+h_1J_1)^2-j\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}Q_{mn}(I_m+h_1J_m)^2} \quad (1)$$

where $A=1$ is the amplitude of the incident wave,

$$P=\frac{F^2ab\sin^2\beta w\sin^2\frac{\pi s}{a}}{\omega\epsilon_0}$$

$$X=\frac{2}{\beta^3}\left(1-\frac{\pi^2}{a^2k_0^2}\right)\sin^2\frac{\pi s}{a}(\cos 2\beta w-1)$$

$$Y=\frac{2}{\beta^3}\left(1-\frac{\pi^2}{a^2k_0^2}\right)\sin^2\frac{\pi s}{a}(2\beta w-\sin 2\beta w)$$

$$Q_{mn}=\frac{c_0}{\alpha_{mn}^3}\left(1-\frac{m^2\pi^2}{a^2k_0^2}\right)\sin^2\frac{m\pi s}{a}\cdot[4\alpha_{mn}w-2(1-\exp(-2\alpha_{mn}w))]$$

$$c_0\begin{cases}=0, & \text{for } n=0; m=1\\ =1, & \text{for } n=0; m>1\\ =2, & \text{for } n>0; m>1\end{cases}$$

$$I_m=\int_{-\pi/2}^{\pi/2}\cos m\alpha\theta\cos\theta\,d\theta =\begin{cases}\pi/2, & \text{when } m\alpha=\frac{2ml}{a}=1\\ \frac{2}{1-m^2\alpha^2}\cos\frac{m\pi\alpha}{2}, & \text{when } m\alpha\neq 1\end{cases}$$

$$J_m=\int_{-\pi/2}^{\pi/2}\cos m\alpha\theta\cos 3\theta\,d\theta =\begin{cases}\pi/2, & \text{when } m\alpha=3\\ \frac{6}{m^2\alpha^2-9}\cos\frac{m\pi\alpha}{2}, & \text{when } m\alpha\neq 3.\end{cases}$$

At resonance the coupling coefficient c_{10} will be real, which implies that the imaginary terms in the denominator of (1) must sum to zero. Thus at resonance the variational method gives

$$c_{10}=\frac{P}{X} =\frac{\beta^2}{2\omega\epsilon_0k_0Z_0\left(1-\frac{\pi^2}{a^2k_0^2}\right)}. \quad (2)$$

But

$$\beta^2=k_0^2-\frac{\pi^2}{a^2} \quad \text{and} \quad k_0Z_0=\omega\mu_0$$

thus at resonance, $c_{10}=0.5=-6$ dB, and this is in exact agreement with Pandharipande's resonance value for c_{10}.

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Comments on "Coupling of Waveguides Through Large Aperture"

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The authors thank Dr. Sangster for his useful comments[1] on their paper [1]. At the time of preparation of the paper it was not known to the authors that there was an unfortunate printing

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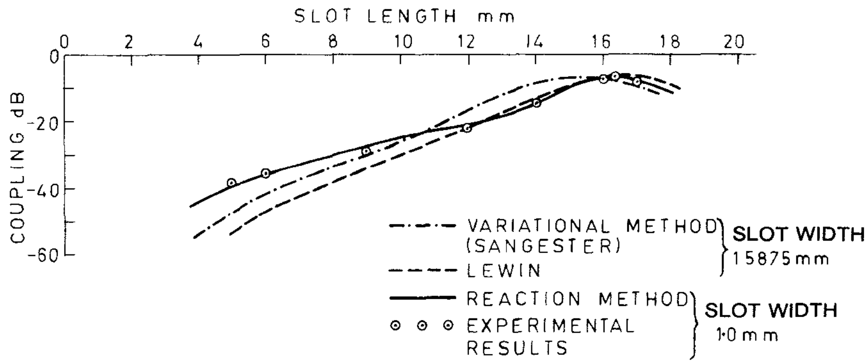

Fig. 1. Variation of coupling with slot length for centered transverse slot in the common broad wall between two guides x = 32 mm.

error in reproduction of eq. (34) of [2]. The authors, however, are still unable to understand the correlation between the theoretical results of Fig. 4 and the comparison between theory and experiment presented in Fig. 7 of [2]. In the modified results of Fig. 2 of the comments no comparison between theory and experiment is presented. The authors, therefore, consider it desirable to present a comparison between the theoretical results corrected for wall thickness and the experimental results on coupling. In this connection it is pertinent to point out that eq. (13) of [1] should read as

$$C_{10}\,\mathrm{dB} = 20 \log \frac{1/\bar{B}}{2\sqrt{1+\frac{1}{\bar{B}^2}}} - 8.686\,\alpha t \tag{1}$$

where $\bar{B}$ is the normalized susceptance of the aperture, α is the attenuation constant in the slot waveguide, and t is the wall thickness of the guide.

Coupling in decibels computed from (1) is presented in Fig. 1, together with the experimental results and theoretical results of Sangster [2] and Lewin [3] corrected for wall thickness $t = 1.58$ mm. The results obtained by the reaction method are found to be in good agreement with the experimental results.

It is worthwhile to point out in this connection that because of the simplifying assumption made in deriving expression (12) of [1], the results on coupling obtained from the self-reaction concept will approach those obtained from asymptotic representation by dipole moments due to Bethe [4], and from quasi-static methods [3], [5] in the limiting case of very thin slots. A careful examination of the analysis presented in the paper [1] would reveal that the width of the slot for which the theoretical results are applicable should, therefore, be assumed to get smaller as the length of the slot is reduced. The apparent discrepancy observed by Dr. Sangster for the slot width of 1.5875 mm can be attributed to this restriction of the equivalent circuit parameter to a one-dimensional problem for small slot length. The equivalent network parameter can also be determined from the self-reaction of the equivalent planar magnetic current. In the latter case the computation involves the evaluation of a double-infinite series—whose convergence also depends upon the slot width. The authors, therefore, do not feel that the reaction method leads to the results which are at variance with those obtained from other theories.

It is perhaps proper to point out that the objective of [1] was to obtain a closed-form expression for the equivalent network parameter of long slots. The only results on the equivalent network parameter with which comparison could be made were those of Levinson and Fredberg [6]. The results obtained by the authors [1] and also those obtained by Levinson and Fredberg [6] exhibit that the resonance takes place for a slot length very near $\lambda/2$. The expression (12) of [1] reveals that the resonance does not take place exactly at $\lambda/2$. A careful examination of the discussion on "correction for the large area of the apertures" presented by Levy [7], however, reveals that the resonance for slots is expected to take place for slot length $= \lambda/2$. The fact that resonance takes place for slot length $= 0.47\lambda$ was obtained by Oliner [8] for slots in a waveguide wall radiating into free space. In the case of a radiating slot, the reactance is contributed by plane waves outside the visible region of the wavenumber plane [9]. For the problem under investigation, the reactance is contributed by the energy storage due to the evanescent modes in the rectangular waveguide. The results for a radiating slot and those for a slot coupling.

The particular advantage of the equivalent circuit approach presented in the paper is that it can be used for the determination of not only the coupling but also the impedance loading in the exciting guide.

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